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LETTER

Diameter variability and strength scatter of elementary flax fibers

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The tensile strength of brittle materials, controlled by the onset of flaw propagation, is commonly characterized by the two-parameter Weibull distribution of strength P_σ [1]

$$P_\sigma(x) = 1 - \exp\left[-\frac{V}{V_0}\left(\frac{x}{\beta}\right)^\alpha\right] \tag{1}$$

where V designates the material volume under stress, V_0 is a normalizing parameter, and α, β stand for Weibull shape and scale parameters, respectively. Due to negligible scatter of diameters of most man-made fibers, fiber length, l, rather than volume is typically applied as a scale variable in the strength distribution function, leading to

$$P_\sigma(x) = 1 - \exp\left[-\frac{l}{l_0}\left(\frac{x}{\beta}\right)^\alpha\right]. \tag{2}$$

However, some variation of diameter along fiber length and/or among fibers is present in inorganic, see, e.g. [2–7], as well as organic fibers of animal [8–11] and plant [12–14] origin. It may need to be reflected not only in the determination of critical stress from fiber test results, but also in the analytical form of the strength distribution function.

The strength of a tension-tested fiber is usually evaluated using load at failure and the average cross-sectional area of the fiber (ASTM D3379 standard), estimated, e.g., via measurements of the fiber diameter at several locations along its gauge length before testing. Determining the cross-sectional area at the fiber fracture location after test (ASTM C1557 standard) can be hampered by shattering of the brittle fibers upon fracture or by localized yielding preceding failure of the more ductile ones. Numerical modeling performed in [15] indicated that the scatter of diameter between fibers should always be taken into account. Analysis reported in [16] suggested that the diameter variation effects become significant if the coefficient of variation of diameter exceeds 0.15 of that of fiber strength. It has been demonstrated experimentally [17] that even at higher diameter dispersion, using the average fiber diameter provided accurate estimates of the Weibull modulus of fiber strength distribution.

In the presence of substantial diameter variation, the fiber strength distribution function can be determined by three basic approaches. The first of them involves evaluation of the failure probability of a differential length element by Eq. 1 with subsequent integration over fiber length. It leads eventually to the strength distribution function of the form of Eq. 2 with a correction factor in the exponent reflecting the effect of variable diameter [7, 13, 18]. The approach is applicable in the case of uniform flaw density with the characteristic flaw size being much smaller than the diameter, and moderate diameter fluctuations. The second approach applies when the transverse size effect of a fiber differs from the longitudinal one; in such a case separate scaling exponents for length and diameter effect on strength are introduced [19, 20]. The third method focuses on determination of the minimum cross-sectional area of the fiber as the most likely location of failure, and relates the strength distribution to the minimum diameter distribution [9, 16]. This approach applies in the case of sufficiently large diameter variation along fiber, when higher stress within low-diameter zone of the fiber can overshadow the effect of flaws elsewhere.

The elementary fibers of flax, being single bast cells, possess a complex, composite internal structure [21] and marked variability in geometrical characteristics [12]. The

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predominant type of mechanical defects routinely observed in flax fibers is kink bands, i.e., distortion zones of the oriented, highly crystalline cellulose microfibrils of the secondary cell wall [21, 22]. Kink bands are shown to serve as the preferential initiation sites of fiber fracture [22, 23] and to reduce fiber strength [21, 24]. The latter effect, however, is relatively mild: three-fold increase of kink band number in fibers did not appreciably affect fiber strength [23, 25], and the average strength of kink band-free fibers exceed only by 20% that of fibers containing kink bands [21]. Furthermore, fragmentation tests during which flax fibers, embedded in polymer matrix, were subjected to virtually uniform strain (and therefore test results should be insensitive to geometrical variations in fiber) yielded higher Weibull shape parameter estimates than fiber tension tests [26]. These observations suggest that not only mechanical defects but also the variability of fiber geometry affects the distribution of flax fiber strength. In this study, the variability of diameter of elementary flax fibers is characterized and its relation with strength distribution discussed.

Assuming that the geometrical irregularity alone is the cause of apparent fiber strength scatter, fiber strength distribution can be derived as follows. Fiber fails when the stress acting on the minimum cross-section of the fiber reaches the intrinsic strength σ_f. The apparent strength of the fiber, determined as the breaking load per average cross-sectional area $\langle S\rangle$ of the fiber, is then given by

$$\sigma = \sigma_f \frac{S_{min}}{\langle S\rangle} \tag{3}$$

where S_{min} designates the minimum cross-sectional area of the fiber. Treating fiber cross-section as circular for simplicity, evaluating $\langle S\rangle$ via the average fiber diameter $\langle d\rangle$ and expressing S_{min} via d_{min}, we obtain $\frac{S_{min}}{\langle S\rangle} = \left(\frac{d_{min}}{\langle d\rangle}\right)^2$. This, in combination with Eq. 3, leads to

$$\sigma = \sigma_f \delta^2 \tag{4}$$

where δ stands for the ratio of minimum and average diameters of given fiber. It follows from Eq. 4 that the distribution function of fiber strength is related to that of δ as

$$P_\sigma(x) = P_\delta\left(\sqrt{x/\sigma_f}\right). \tag{5}$$

Diameter variation of the fibers produced by Ekotex (Poland) was examined. Elementary flax fibers were carefully manually separated from the technical fibers supplied by the producer. For ease of handling, fiber ends were glued onto a paper frame so that free fiber length amounted to 5 mm. Nine such specimens were prepared. Olympus BX51 microscope was used for optical inspection of the fibers. Several digital pictures of consecutive fiber zones, covering the entire length of each fiber, were taken by a CCD camera attached to the microscope. The pictures were processed by a purpose-written code for image analysis enabling fiber diameter measurements at predefined intervals. In each picture, the apparent fiber diameter was measured at c.a. 2 μm intervals within a fragment of $l = 0.5$ mm length. Then, for each fiber fragment, using these measurements, the minimum diameter, d_{min}, and the average diameter, $\langle d\rangle$, were determined and their ratio $\delta = d_{min}/\langle d\rangle$ calculated. The empirical distribution function of the normalized minimum diameter, based on δ values of all the fiber fragments characterized, is plotted in Fig. 1 in Weibull co-ordinates. It is seen that the empirical data appear reasonably close to the Weibull two-parameter distribution

$$P_\delta(x) = 1 - \exp\left[-\frac{l}{l_0}\left(\frac{x}{\delta_0}\right)^m\right]. \tag{6}$$

Setting $l_0 = 0.5$ mm for simplicity, the parameters of distribution Eq. 6 were determined by approximating the empirical distribution in Fig. 1 as $m = 8.8$ and $\delta_0 = 0.89$.

If diameter variability is controlling the apparent fiber strength, it follows from Eqs. 5 and 6 that the distribution function of strength reads as

$$P_\sigma(x) = 1 - \exp\left[-\frac{l}{l_0}\left(\frac{x}{\sigma_f \delta_0^2}\right)^{m/2}\right]. \tag{7}$$

Mean fiber strength, according to distribution function Eq. 7, is

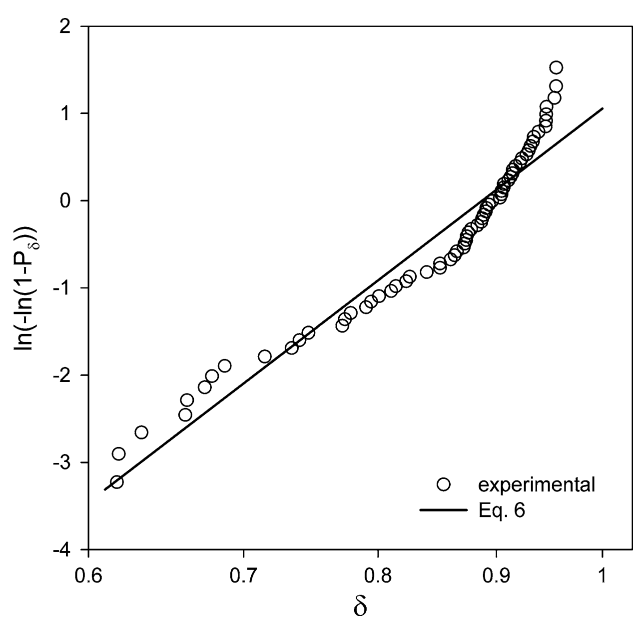

Fig. 1 Distribution of the normalized minimum diameter of flax fiber fragments, plotted in Weibull co-ordinates

$$\langle \sigma \rangle = \sigma_{\mathrm{f}} \delta_0^2 \left(\frac{l}{l_0} \right)^{-2/m} \Gamma\left(1 + \frac{2}{m}\right). \tag{8}$$

Equation 7 is in qualitative agreement with strength data of elementary flax fibers of the same origin and 10 mm gauge length, shown in [27] to possess the Weibull two-parameter distribution. The estimate of the intrinsic fiber strength, obtained by Eq. 8 using the experimental average strength from [27], amounted to $\sigma_{\mathrm{f}} = 1980$ MPa. This value only slightly exceeds the mean strength of kink band-free elementary fibers reported in [21], although σ_{f} should constitute an upper limit for fiber strength. Moreover, the shape parameter value $m/2 = 4.4$ of the theoretical strength distribution Eq. 7 based on minimum diameter measurements is not consistent with the shape parameter of fiber strength distribution Eq. 2 evaluated at $\alpha = 2.7$ in [27].

One can conclude that fiber diameter variation is not the primary mechanism determining the strength distribution of the elementary flax fibers considered. The critical limiting factor of strength apparently was mechanical damage of the fibers in the form of kink bands [28]. Nevertheless, irregularities of fiber geometry may play an important role in determining the strength of undamaged flax fibers that, depending on the growth conditions and processing method, can constitute a sizable fraction of the fibers produced. To derive the strength distribution of such a fiber batch, the combined effect of fiber geometry and mechanical defects on strength has to be considered, which is a subject for further research.

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